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Two-Fluid Modeling Versus Mechanistic Approach and Lift Effects in Bubbly Sheared Flows[1]

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The behavior of bubbles in a shear flow is experimentally studied. The lift force experienced by a single bubble in constant shear is determined. It proves to be roughly equal to a half of the virtual mass coefficient up to Reynolds numbers of the order of 2000. Besides, the void—migration taking place inside a bubbly boundary layer on a flat plate is studied and shown to be associated with an actual lateral deflection of the bubbles only under certain conditions. Finally, the link between the lagrangian description of the motion and the two-fluid model is qualitatively discussed.

KEY WORDS: bubbly flows; lift effects; void migration.

1. INTRODUCTION

There are two vastly different but equally general and mathematically exact pictures of a dispersed two-phase flow of Newtonian fluids:

(i) that of the two original fluids exchanging momentum at the interfaces—their motions are governed by the Navier–Stokes equations; and

(ii) that of two equivalent fictitious coexisting phases whose momentum interaction is volumetrically distributed, as a result of the ensemble averaging performed when deriving (ii) from (i).

The *microscale description* (i) is, of course, totally impractical. The *two-fluid description* (ii), on the other hand, is much more tractable if the governing set of equations can be closed, i.e., if the unknown terms, among

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which are the momentum interaction terms, can be properly modeled. Loosely speaking, the latter description is widely used by engineers who need numerical codes, since it provides a "simple" and systematic treatment of a wide variety of configurations, sometimes at the expense of physical relevance. In order to avoid such shortcomings, the academic world very often will go back to some highly idealized version of the original microscale flow, hoping that such a "mechanistic approach" will provide a good closure law for the momentum interaction, thus nicely closing the gap between the real and averaged descriptions (i) and (ii)—obviously at the expense of generality.

One way of achieving this is to resort to an intermediate lagrangian description, which consists in tracking each individual inclusion of mass m^i and applying Newton's law to its center of mass whose velocity, initial, and current positions with respect to a given frame of reference are, respectively, denoted $\vec{V}_i$, $\vec{a}_i$, and $\vec{X}_i(t)$:

$$m_i \frac{\partial \vec{V}_i}{\partial t}(\vec{a}_i, t) = \vec{F}_i(\vec{a}_i, t) \tag{1}$$

However rigorous Eq. (1) might look, it will be physically realistic only if the inclusions are small enough, compared to some typical length scale and if the hydrodynamic force $\vec{F}_i$ exerted on it by the continuous phase can be reasonably well modeled—which implies, at least, an approximate integration of the local and instantaneous flow equations, in its vicinity. Eventually, the constitutive equation required for the momentum interaction terms appearing in the two-fluid model will be a mere transposition to the average flows of the local and instantaneous form of $\vec{F}_i$. That such an approach is not always consistent, has been pointed out by Simonin [1] and recently discussed rigorously for slightly nonuniform suspensions by Lhuillier [2].

In what follows, the authors will illustrate some of these difficulties using the experimental and numerical results recently obtained by several coworkers or former students (Marié, Naciri, Moursali, and Petersen) as they were studying lift effects experienced by air bubbles in shear flows and the void migration occurring in a bubbly turbulent boundary layer on a vertical flat plate.

2. VOID MIGRATION IN A BOUNDARY LAYER AND DEFLECTION OF BUBBLES TOWARD THE WALL

According to the foregoing, there are two complementary ways of investigating the void distribution inside a turbulent boundary layer developing on a flat plate immersed in a uniform vertical upward bubbly flow. The first one, which corresponds to the *two-fluid model viewpoint*, consists in the determination of the local *void fraction*; the second one, in keeping with the *Lagrangian viewpoint*, consists in the tracking of the individual bubbles. Such a program was achieved and partly reported by Moursali et al. [3] using the experimental setup shown in Fig. 1. The detailed description of the hydrodynamic tunnel is given by Lance and Bataille [4]. The upstream liquid mean velocity U_L and void fraction α were uniform, while the mean bubble diameter D_B was approximately constant (~ 4 mm). The void fraction and the mean number of bubbles passing through a given point per unit time—hereafter called bubble frequency F_B—were measured with an optical probe and the visualization of the motion of the bubbles was performed with a high-speed videocamera (200 frames/s).

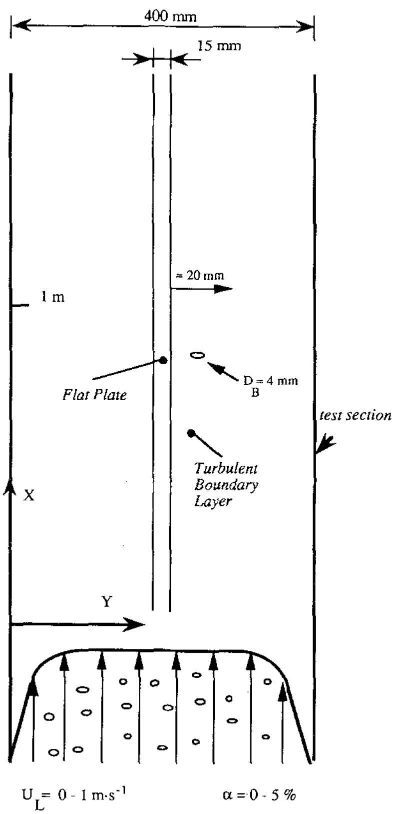

Fig. 1. Sketch of the experimental setup.

Typical void fraction profiles at a given representative station ($X = 1m$) are shown in Fig. 2 for three upstream void fractions. The void fraction exhibits a sharp relative or absolute maximum at a distance of the order of the mean radius of the bubbles and asymptotically recovers its free stream value. The void peaking phenomenon observed here in a comparatively simple situation is not surprising in view of the findings of a number of authors dealing with upward pipe flow [5]. One may wonder, however, whether the sharp increase in α at the wall should be associated with an increase in the bubble frequency F_B and therefore with an actual void migration or with the deceleration of the bubbles which takes place at the wall. Indeed, the void fraction is of the form

$$\alpha \propto \frac{F_B D_B}{U_B} \tag{2}$$

where U_B is the average velocity of the bubbles. As a matter of fact, the evolution of the bubble frequency at the peak F_{BP}, as a function of its free stream value F_{BE} (see Fig. 3), clearly indicates that void migration does not systematically occur: no net statistical deflection of the bubbles takes place, for example, at relatively high values of the void fraction.

The same conclusions can be drawn qualitatively from inspection of the videofilms, which show that a significant number of bubbles undergo a violent deflection toward the wall, depending on the operating conditions. The same cinematographic evidence also suggests that it is mainly when the bubbles are small [$D_B \lesssim 3.5$ mm] that they migrate to the wall, whereas they are hardly deflected when they are larger. That the diameter should play such a role is not totally unexpected since the deflection of a given bubble in a highly sheared turbulent flow can be controlled only by

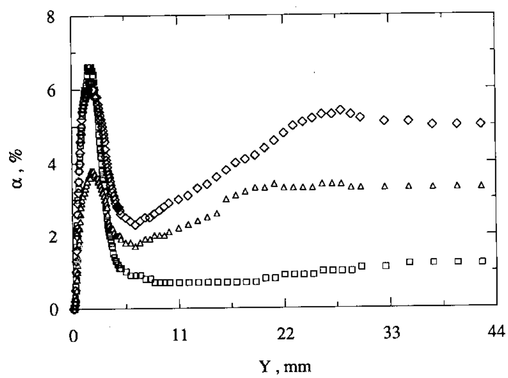

Fig. 2. Void fraction profiles at $X = 1m$ ($U_L = 1\ \mathrm{m \cdot s^{-1}}$).

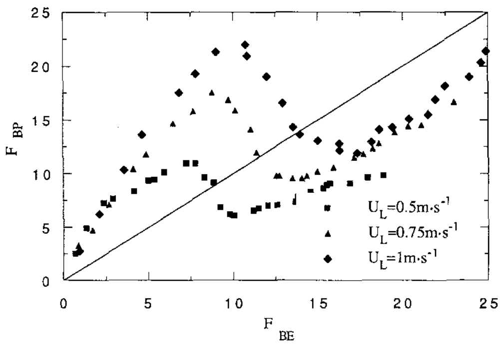

Fig. 3. Bubble frequency at the peak location versus the external frequency at $X = 1m$.

its deformation and associated wake modification and by its interaction with the surrounding turbulent structures.

The foregoing experimental findings raise the following question: Can the lift force exerted on a single bubble in a constant shear flow account for the observed migration?

3. LIFT COEFFICIENT OF AN ISOLATED MILLIMETRIC BUBBLE IN A SHEAR FLOW

In order to determine the lift coefficient C_L of a single millimetric bubble in a constant shear flow, a small volume of air is injected into a

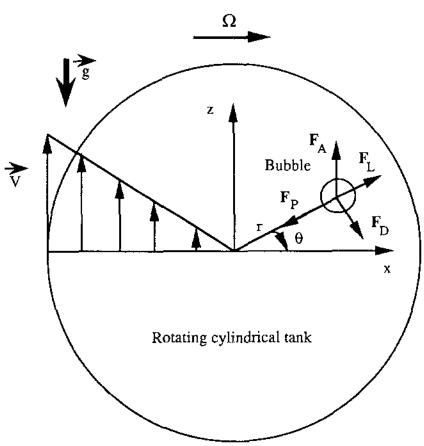

Fig. 4. Lift force experienced by an air bubble in a shear flow.

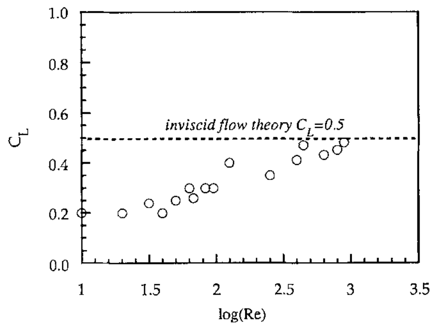

Fig. 5. Lift coefficient as a function of the Reynolds number.

horizontal circular cyclinder containing distilled water and rotating uniformly about its axis at a rate ω. It eventually reaches an equilibrium position defined by its polar coordinates r and θ, under the action of the buoyancy force and the lift, drag, and virtual mass forces associated with the shear flow generated (Fig. 4). Using the equation proposed by Auton et al. [6] for the motion of a bubble, it is straightforward to show that equilibrium requires that

$$2C_{\mathrm{L}} - C_{\mathrm{VM}} = 1 - (g/\omega^2)\frac{\sin\theta}{r} \tag{3}$$

where C_{VM} stands for the vitual mass coefficient. Figure 5 shows that $C_{\mathrm{L}} \sim \frac{1}{2} C_{\mathrm{VM}}$ for Reynolds numbers ranging from 10 to 2000 [7, 8].

4. COMPARISON BETWEEN NUMERICAL ESTIMATES AND EXPERIMENTAL DATA

The flat plate problem was solved numerically using a code described elsewhere [9, 10], based on the two-fluid model. The lateral momentum transfer from the liquid to the gas, expressed in terms of the average velocity fluids, was directly inferred from the lift force derived by Auton et al. [6] for a single bubble, with the lift coefficient determined in the previous section. It can be seen in Fig. 6 that the results obtained compare reasonable well with the experimental data, whatever the modeling of the Reynolds stress tensor. On the other hand, all attempts made at predicting the observed individual trajectories of the deflected bubbles, using the very same "average" lift force, hopelessly failed.

There is no paradox, however, since the void migration process is the average result of the statistical behavior of a population of bubbles. As for

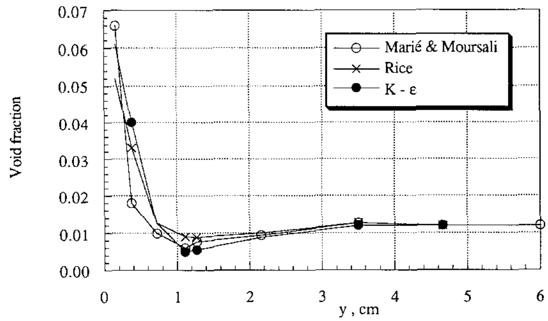

Fig. 6. Comparison between measured and calculated void fraction profiles for different turbulent models.

the discrepancy concerning the dynamics of a given bubble, it could probably be removed if the lift force was expressed in terms of the local and instantaneous velocity fields, rather than the average ones. In the case of a sudden expansion, however, the very same numerical code gives very poor results [10], corroborating our belief that a lot more work remains to be done in order to bridge the gap between the ensemble averages occurring in the two-fluid model and the physical quantities which naturally appear in the Lagrangian of a highly sheared dispersed flow.

5. FROM LAGRANGIAN TO EULERIAN FORMULATION

By way of conclusion, it is worthwhile to derive a two-fluid model from the Lagrangian description in the extremely crude case of N identical nondeformable inclusions of mass m, whose individual motions are mere translations. Under such conditions, the characteristic function $\chi_i(x, t)$ of the ith inclusion obeys the following equation:

$$\frac{\partial \chi_i}{\partial t} + \vec{V}_i \cdot \overrightarrow{\nabla \chi} = 0 \tag{4}$$

Using Eqs. (1) and (4), one can readily show that

$$\frac{\partial}{\partial t}(\chi_i \vec{V}_i) + \vec{\nabla} \cdot \chi_i(\vec{V}_i \otimes \vec{V}_i) = \chi_i \frac{\vec{F}_i}{m} \tag{5}$$

Summing over all particles, taking ensemble averages—denoted by bracketed quantities—and defining the volumetric concentration α_{d}, the mean velocity V_{d} of the dispersed phase, and its fluctuation V_i' by

$$\alpha_{\mathrm{d}} = \left\langle \sum_{i=1}^{N} \chi_i \right\rangle; \qquad \alpha_{\mathrm{d}} \vec{V}_{\mathrm{d}} = \left\langle \sum_{i=1}^{N} \chi_i \vec{V}_i \right\rangle; \qquad \vec{V}_i' = \vec{V}_i - \vec{V}_{\mathrm{d}} \tag{6}$$

and the familiar mass and momentum conservation equations of the two-fluid model are recovered:

$$\frac{\partial \alpha_d}{\partial t} + \vec{\nabla} \cdot (\alpha_d \vec{V}_d) = 0 \tag{7}$$

$$\frac{\partial}{\partial t}(\alpha_d \vec{V}_d) + \vec{\nabla} \cdot (\alpha_d \vec{V}_d \otimes \vec{V}_d) + \left\langle \sum_{i=1}^{N} \chi_i (\vec{V}_i' \otimes \vec{V}_i') \right\rangle = \left\langle \sum_{i=1}^{N} \chi_i \frac{\vec{F}_i}{m} \right\rangle \tag{8}$$

If, for the sake of simplicity, all hydrodynamic forces but for the drag are ignored, one may write, at low Reynolds numbers,

$$\vec{F}_i = k[\vec{U}(\vec{X}_i, t) - \vec{V}_i(t)] \tag{9}$$

where $\vec{U}(\vec{X}_i, t)$ is the velocity of the continuous liquid phase at the center of mass $\vec{X}_i$ of the inclusion. Such a force, which is based on the actual local and instaneous velocities of both phases, will account for the experimentally observed strong accelerations of the inclusions. If the topology of the continuous phase is defined by its characteristic function χ_c, its average velocity $\vec{V}_c$ is given by

$$(1 - \alpha_d)\vec{V}_c = \langle \chi_c \vec{U}(\vec{x}, t) \rangle \tag{10}$$

and the right-hand side of Eq. (8) may be shown to have the following form:

$$\left\langle \sum_{i=1}^{N} \chi_i \frac{\vec{F}_i}{m} \right\rangle = \frac{k}{m} \alpha_d (\mathbf{V}_c - \mathbf{V}_d) + \frac{k}{m} \left\langle \sum_{i=1}^{N} \chi_i [\vec{U}(\vec{X}_i) - \vec{V}_c] \right\rangle \tag{11}$$

The momentum–interaction term appearing in the two-fluid model can accordingly be split into two contributions. The first one is the familiar Stokes relationship applied to the average velocity fields, commonly employed when closing the two-fluid model. The second one, however, which is reminiscent of Simonin's suggestion [1], is linked to the difference between the actual velocity of the continuous phase and its average. It is expected to vanish in uniform situations but might contribute significantly to the average motion of the dispersed phase in those regions of the flow where the distribution of particles is highly nonuniform—such as large-scale vortical motions of the continuous phase.

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